
ABSTRACT

In this study, a finite element analysis was used to design composite drive shafts incorporating carbon and glass fibers within an epoxy matrix. A configuration of one layer of carbon-epoxy and three layers of glass-epoxy with 0°, 45° and 90° was used. The developed layers of structure consists of four layers stacked as 1/2p45_glass=45_glass=0_carbon=90_glass. The results show that, in changing carbon fibers winding angle from 0° to 90°, the loss in the natural frequency of the shaft is 44.5%, while, shifting from the best to the worst stacking sequence, the drive shaft causes a loss of 46.07% in its buckling strength, which represents the major concern over shear strength in drive shaft design.

KEYWORDS: Drive shafts, finite element analysis, torsional vibration systems.

INTRODUCTION

Drive shafts for power transmission are used in many applications, including cooling towers, pumping sets, aerospace, structures, and automobiles. There is unique value for the shaft's inner radius because the outer radius is constrained by the space under the car cabin. Metallic drive shafts have limitations of weight, low critical speed and vibration characteristics. When the length of a steel drive shaft is beyond 1500 mm [3], it is manufactured in two pieces to increase the fundamental natural frequency, which is inversely proportional to the square of the length and proportional to the square root of the specific modulus. The nature of composites, with their higher specific elastic modulus (modulus to density ratio), which in carbon/epoxy exceeds four times that of aluminium, enables the replacement of the two-piece metal shaft with a single component composite shaft which resonates at a higher rotational speed, and ultimately maintains a higher margin of safety. A composite drive shaft offers excellent vibration damping, cabin comfort, reduction of wear on drive train.

Composite drive shafts have proven that they can solve many automotive and industrial problems that accompany the usage of the conventional metal ones. Components and increased tire traction. In addition, the use of single torque tubes reduces assembly time, inventory cost, maintenance, and part complexity. The first application of composite drive shafts to automobiles was developed by Spicer U-joint divisions of the Dana Corporation for the Ford Econoline van models in 1989 [3]. Polymer matrix composites such as carbon/epoxy or glass/epoxy offer better fatigue characteristics because micro cracks in the resin do not freely propagate as in metals, but terminate at the fibers. Generally, composites are less susceptible to the effects of stress concentration, such as are caused by notches and holes, compared with metals [4]. The filament winding process is used in the fabrication of composite drive shafts. In this process, fiber tows wetted with liquid resin are wound over a rotating male cylindrical mandrel. In this technique, the winding angle, fiber tension, and resin content can be varied. Filament winding is relatively inexpensive, repetitive and accurate in fiber placement [5]. An efficient design of composite drive shaft could be achieved by selecting the proper variables, which are specified to minimize the chance of failure and to meet the performance requirements. As the length and outer radius of drive shafts in automotive applications are limited due to spacing, the design variables include the inside radius, layers thickness, number of layers, fiber orientation angle and layer stacking sequence. In the optimal design of the drive shaft, these variables are constrained by the lateral natural frequency, torsional vibration, torsional strength and torsional buckling of the shaft. In this study, another constraint is added that relates to torsional fatigue and selection of the stacking sequence. The ability to tailor the elastic constants in composites provides numerous alternatives for the variables to meet the desired stability and strength of the structure. At first, the fibers are selected to provide the best stiffness and strength, together with their cost. Indeed, it is the best choice

to use carbon fibers in all layers to achieve desired stability. However, due to the cost constraint, a hybrid of layers of carbon–epoxy and E-glass–epoxy was introduced. It is evident that the fiber orientation angle dictates the maximum bending stiffness, in turn leading to the maximum natural frequency in bending. In this design, the fibers were arranged longitudinally at the zero angle with respect to the shaft axis. On the other hand, the angle of $\pm 45^\circ$ was used to obtain the maximum shear strength, while 90° was the best for buckling strength. The main goal was to achieve the minimum weight while adjusting the parameters in order to meet a sufficient margin of safety. The safety criteria specify that a critical speed (natural frequency) must be higher than the operating speed, a critical torque must be higher than the ultimate transmitted torque and a nominal stress (the maximum at fiber direction) must be less than the allowable stress after applying the failure criteria. In the shaft design, the shear strength could be increased by increasing the diameter of the shaft. However, the crucial parameter to consider was the buckling strength. The variable of the laminate thickness has a big effect on the buckling strength and slight effect on the natural frequency in bending. A discrete variable optimization algorithm could be employed for optimization of layer thickness and orientation. Vijayarangan *et al.* [6] used a Genetic Algorithm, and Rastogi [3] use GENESIS/I-DEAS optimizers for the optimization of variables in the design of a drive shaft for automotive applications. In other work, Darlow and Creonte [7] employed the general-purpose package, OPT (version 3.2), in optimizing the lay-up of a graphite–epoxy composite drive shaft for a helicopter tail rotor.

DESIGN PROCEDURE

The material properties of the drive shaft were analyzed with classical lamination theory. The theory treats with the linear elastic response of laminated composite under plane stress, and it incorporates the Kirchhoff-Love assumption for bending and stretching of thin plates [8]. From the properties of the composite materials at given fiber angles, the reduced stiffness matrix can be constructed. The expressions for the reduced stiffness coefficients (Q_{ij}), in terms of standard material constants, are as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12},$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad \text{and} \quad \nu_{21} = \frac{E_2}{E_1}\nu_{12},$$

where E is modulus of elasticity, G is modulus of rigidity and ν is Poisson's ratio.

The next step is to construct the extensional stiffness matrix $[A]$. This matrix is the summation of the products of the transformed reduced stiffness matrix $\frac{1}{2}Q_k$ of each layer and the respective thicknesses, represented as:

$$[A] = \sum_{k=1}^N [Q]^k (z_k - z_{k-1}).$$

The matrix, $[A]$, is in $(\text{Pa} \cdot \text{m})$, and the thickness of each ply is calculated in reference to their coordinate location in the laminate. The A matrix is used to calculate E_x and E_h , which are the average moduli in the axial and hoop directions, respectively

$$E_x = \frac{1}{t} \left[A_{11} - \frac{A_{12}^2}{A_{22}} \right], \quad E_h = \frac{1}{t} \left[A_{22} - \frac{A_{12}^2}{A_{11}} \right].$$

Buckling torque

Since the drive shaft is considerably long, thin and hollow, there is a possibility that it may buckle. The expression of the critical buckling torque for thin-walled orthotropic tubes [9] is given as:

$$T_{cr} = (2\pi r^2 t) (0.272) [E_x E_h^3]^{1/4} \left(\frac{t}{r} \right)^{3/2}.$$

Here, r is the mean radius and t is the total thickness. It is obvious that the stiffness modulus in hoop direction (E_h) plays most substantial role in increasing the buckling resistance. The safety factor is defined as the ratio of the buckling torque to the ultimate torque.

Literal bending natural frequency

The drive shaft is designed to have a critical speed of 60 times larger than the natural frequency of the rotational speed. If these become coincident, a large amplitude vibration (whirling) will occur. The drive shaft is idealized as either a simply-supported or a pinned–pinned beam. The lowest natural frequency expression [10,11] is given as:

$$f_n = \frac{\pi}{2} \sqrt{\frac{gE_x I}{WL^4}}$$

where g is the acceleration due to gravity, W is the weight per unit length, L is the shaft length and I is the second moment of inertia given, for a thin-walled tube, as:

$$I_x = \frac{\pi}{4} (r_o^4 - r_i^4) \approx \pi r^3 t$$

Here, r_o is an outer radius, and r_i is an inner radius.

Load carrying capacity

The composite drive shaft is designed to carry the torque without failing. The torsional strength, the torque at which the shaft will fail, is directly related to the laminate shear strength through:

$$T_s \approx 2\pi r^2 t s_l$$

Here, T_s is the failure torque, s_l is the in-plane shear strength of the laminate, r_m is the mean radius and t is the thickness. The same formula is used in the laboratory after a torsion tube test to determine the shear modulus and shear strength of materials. Since the laminate is assumed to have failed according to the first ply failure convention, the maximum-stress failure criterion could be used after finding the in-plane stresses at every ply to specify the safety factor for torque transmission capacity. Again, the first steps are to construct the transverse of the extensional stiffness matrix $[A]$ and then solve for the overall strains. Once complete, the stresses in each layer can be examined by transforming these stresses into the XZY_l direction of fibers at each layer. The layers of fiber direction $\pm 45^\circ$ are of special concern since they have a substantial contribution to the load carrying capacity. The A_{-1} matrix, multiplied by the laminate thickness and the resultant forces matrix, N , gives the resultant strain as follows:

$$\{\epsilon\} = A^{-1} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = A^{-1} \begin{Bmatrix} 0 \\ 0 \\ N_{xy} \end{Bmatrix}, \quad N_{xy} = \frac{T}{2\pi r^2}$$

Here, the axial force is $N_x = 0$, the centrifugal force, N_y , is neglected and N_{xy} is the resultant shear force. The torque, T , is the peak torque if the design involves fatigue considerations [12]. The resultant strains are transformed to the fiber direction by multiplying these strain matrices by the transformation matrix. From this, the plane stress transformations can be obtained.

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$= \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Torsional frequency

The torsional frequency is another parameter and is directly related to the torsional stiffness (T/u), where u is the angle of twist and T is the applied torque. The frequency of torsional vibration can be presented as:

$$f_t = \frac{1}{2\pi} \sqrt{\frac{K}{I_m}}$$

where K is the torsional spring rate, and is equal to the torsional stiffness, and I_m is the mass moment of inertia at propeller. For a given geometry in a specific drive shaft, the torsional stiffness is directly related to the modulus of rigidity (G_{xy}) as follows [2]:

$$K = \frac{T}{\phi} = \frac{G_{xy}J}{L},$$

where J is the polar moment of inertia and L is the length. The shear modulus can be tailored to its maximum value by orienting the fibers at an angle equal to 45° . In some applications, like racing cars, less torsional stiffness is required [2]. The shear modulus can be directly obtained from the extensional stiffness matrix $[A]$, by dividing the shear stiffness component, A_{66} , by the total thickness of the drive shaft as follows:

$$G_{xy} = \frac{A_{66}}{t}.$$

A practical application of torsional vibration systems is in engines. These engines have damping (source of energy dissipation) in the crankshafts (hysteresis damping), and damping in torsional vibration (in propellers). Since the damping present is normally small in magnitude, it can be neglected when determining the natural frequency [13].

Finite element analysis of drive shaft

Finite element models of the drive shaft were generated and analyzed using LUSAS (version 13.5-7) commercial software. A three-dimensional model was developed and meshed with three dimensional thick-shell elements (QTS8). This degenerate continuum element is capable of modeling warped configurations, accounting for varying thicknesses and supporting the definition of anisotropic and composite material properties. Since it is quadrilateral, it uses an assumed strain field to define transverse shear, which ensures that the element does not lock when it is thin. Such elements can accommodate a broader range of curved geometries than other element types can [14]. A cylindrical local coordinate system was defined in order to align the material axis of the layup, and to apply fixities and load cases. Eigenvalue linear buckling analysis was performed to define the critical buckling torque. The output from this analysis is a factor that is multiplied by the applied load to determine the critical buckling load. The linear analysis is considered satisfactory in comparison with nonlinear analysis due to the fact that cylindrical shells under torsion are less sensitive to imperfections [15]. In this study, the position of the buckling region along the axial length of the shaft was detected as being shifted towards the end of the shaft when a nonlinear analysis was performed. Modal analysis is a technique used to analyze structures dominated by global displacement, such as in vibration problems. It was used to define the natural frequency of the drive shaft. The Eigenvectors resulting from the Eigenvalue analysis are the modes of the buckling deformation and the natural frequency in bending, as presented in Figs.1 and 6. A composite drive shaft design example, presented by Swanson [12], was taken as a reference model for all analyzes. In this example, a shaft of length 1730 mm, mean radius 50.8mm and consisting of three layers of ($\pm 45^\circ$, 90°) glass-epoxy and 0° carbon-epoxy layer was used. The ultimate torque was 2030 Nm and the minimum natural frequency in bending is 90 Hz. The material properties are listed in Table 1.

Specimens fabrication

Four layers of carbon/epoxy, glass/epoxy and a hybrid of both were wrapped around aluminium tubes of length equal to 216mm and outside diameter equal to 12.7 mm. For easily removing the aluminium tubes, thin film of oil was formed then a thin plastic sheet wrapped around. The epoxy impregnated carbon and glass fabrics had been wrapped with plastic sheet at outside surface for the purpose of producing smooth surface. These tubes are removed after curing under room temperature. The ends of the specimens were reinforced by the winding of carbon or glass fibers tows used in filament winding. The stacking of these specimens is as follows:

1. $[\pm 45]_4$ All layers are of glass/epoxy.
2. $[\pm 45]_4$ All layers are of carbon/epoxy.
3. $[90]_4$ All layers are of glass/epoxy.
4. $[90]_4$ All layers are of carbon/epoxy.
5. $[(\pm 45)_2 \text{ glass}/(90)_2 \text{ carbon}]$.
6. $[(\pm 45)_2 \text{ carbon}/(90)_2 \text{ glass}]$.

Woven roving Fabric fibers used in both $[0/90]$ and $[\pm 45]$ lay-up. The thicknesses of the composites were measured to be: Carbon/epoxy layer thickness = 0.35 mm and glass/epoxy layer thickness = 0.37 mm.

RESULTS AND DISCUSSION

Effect of fiber orientation angle on natural frequency

The vibration problem is described by a set of equations, and there is a natural vibration mode for every equation that can be extracted by using an eigenvalue extraction analysis. The displacement behavior dominating any structure subject to vibration is global; therefore, modal analysis is utilized in these types of problems. In modal analysis, the model of the drive shaft does not need a fine mesh because the stress output is not required. Additionally, there is no requirement to input an applied load, because the natural frequency is only a function of mass and stiffness. The ends of the drive shaft model were modeled as simply-supported, and the boundary condition was varied until the value of the natural frequency became nearly coincident with that presented by a reliable example. It was recognized that the end support conditions must also be applied to the edges of the drive shaft. For simplicity, the contact area between the shaft tube and the yoke joint, as well as the joint itself, were not considered in the calculations of the natural frequency.

In any structural design with vibration concerns, only the first mode is of concern for engineering applications.

Fig. 1 presents the shape of the first bending mode based on natural frequency. Fig. 2 shows a set of the first six natural frequencies in bending. The drive shaft specified in the previous section was used to investigate the effect of fiber orientation angle on the natural frequency. This structure consists of four layers stacked as $[\frac{1}{2}p45_glass= _45_glass=0_carbon=90_glass]$. From Figs. 3 and 4, it is clear that the fibers must be oriented at zero degrees to increase the natural frequency by increasing the modulus of elasticity in the longitudinal direction of the shaft. This explains why the carbon fibers, with their high modulus were oriented at the zero angle. In Fig. 3, despite the configuration [0, 0, 90, 0] resulting in the highest natural frequency, it is not a good selection when an optimization with other parameters, such as buckling resistance and fatigue strength, is made. From Fig. 4, the drive shaft loses 44.5% of its natural frequency when the carbon fibers are oriented in the hoop direction at 90_ instead of 0_. The cost factor plays a role in selecting only one layer of carbon/epoxy. The analysis was conducted on comparatively thin composite tubes, and shows that the behavior of the thinner tube is different. Specifically, the critical speed and the natural frequency did not increase as the orientation angle approached the value of zero. As seen in Fig. 5, three models of the same material (carbon/epoxy) and different thicknesses were constructed. It was found that the critical speeds for all models were the same when the fibers of all layers were oriented at 38–90_. The fiber angle of 38_, or 37_, as mentioned in the literature (Herakovich, 1998) imparts special properties, since, at this angle, unidirectional off-axis tubes under pure torque loading exhibit the maximum coupling between shear strain and axial strain. The axial strain reaches as much as 50% of the shear strain. However, from this figure, it is clear that, for the tubes of smaller thickness, the membrane stress plays an effective role in the lateral stiffness of the tube. At 38_, the torque coupling coefficient (nT_e) is at the maximum, and hence the axial strain is at the maximum. This directly leads to the highest bending stiffness, implying a higher natural frequency. The stacking sequence has no effect on the natural frequency because the matrix form of the equation of dynamic equilibrium for an elastic body only contain stiffness and mass matrices when no damping and external forces are applied. The mass matrix is a function of the total density and the absence of loads make the stacking sequence irrelevant.

Effect of fiber orientation angle on buckling torque

A linear eigenvalue buckling analysis was conducted to estimate the maximum torque that can be supported prior to losing stability. In this analysis, the specified load must be closer to the collapse load in order to obtain accurate results. The output from the analysis is a factor that can be multiplied by the actual magnitude of the applied load in order to obtain an estimate of the critical torque. Fig. 6 presents the contour of maximum shear stress, and the deformed shape after linear eigenvalue analysis.

Effect of layers stacking sequence on buckling torque

The stacking sequence of the layers has an effect on the buckling strength. Although the [A] matrix is independent of the stacking sequence, both the [B] and [D] matrices are dependent upon it. The drive shaft buckled when its bending stiffness along the hoop direction could not support the applied torsion load. This normal bending stiffness is correspondent to the component, D₂₂, of the bending stiffness matrix [D]. Therefore, the value of D₂₂ specifies the buckling strength. Fig. 9 presents the effect of stacking sequence on the buckling strength and it is concluded that the best case scenario stacking sequence is [45/_45/0/90], and the worst case scenario is [0/90/_45/45]. Table 2 shows the correspondent D₂₂ components for five laminates with different

stacking sequences. The best stacking offers a buckling torque of 2303.1 Nm and the worst stacking offers a torque of 1242 Nm, with a loss in buckling resistance capability equal to 46.07%.

Effect of coupling between the twisting moment and normal curvature

The twisting moment resulted from a pure torque loading coupled with a normal curvature in terms of the components D_{16} and D_{26} in the bending stiffness matrix $[D]$. The D_{16} component represents the curvature in the longitudinal direction, and as its value increases the drive shaft tends to bend, and its natural frequency in bending decreases. The coupling between the twisting moment and the normal curvature in the hoop direction can be directly related to the coefficient of mutual influence (g), which is a normal shear coupling. One form of this coupling is:

$$g_{xy} = \frac{1}{4} \frac{e_{xy}}{c_{xy}} \frac{S_2}{S_6}$$

Here, e_y is strain in y direction and c_{xy} is the shear stress.

This coefficient represents the radial strain resulting from a torque loading, and it may have a negative or positive value. If the sign is positive, the diameter of the cross-section tends to decrease.

CONCLUSIONS

The present finite element analysis of the design variables of fiber orientation and stacking sequence provide an insight into their effects on the drive shaft's critical mechanical characteristics and fatigue resistance. A model of hybridized layers was generated incorporating both carbon–epoxy and glass–epoxy. Buckling, which dominates the failure modes, has a value does not increase regularly with increasing the winding angle. For the worst stacking sequence, the shaft loses 46.07% of its buckling strength compared to what it achieves with the best stacking sequence. On the other hand, the stacking sequence has an obvious effect on the fatigue resistance of the drive shaft.

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